

**Instructions:** Complete each of the following exercises for practice.

1. Use Stokes's Theorem to evaluate  $\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$  for  $S$  oriented upward.
  - (a)  $\mathbf{F}(x, y, z) = \langle x^2 \sin(z), y^2, xy \rangle$ ,  $S$  the part of the paraboloid  $z = 1 - x^2 - y^2$  above the  $xy$ -plane
  - (b)  $\mathbf{F}(x, y, z) = \langle ze^y, x \cos(y), xz \sin(y) \rangle$ ,  $S$  the hemisphere of  $x^2 + y^2 + z^2 = 16$  with  $y \geq 0$
  - (c)  $\mathbf{F}(x, y, z) = \langle \arctan(x^2 y z^2), x^2 y, x^2 z^2 \rangle$ ,  $S$  the cone  $x = \sqrt{y^2 + z^2}$  with  $x \leq 2$
  - (d)  $\mathbf{F}(x, y, z) = \langle xyz, xy, x^2 yz \rangle$ ,  $S$  the boundary of the cube  $[-1, 1]^3$  minus the bottom face
  - (e)  $\mathbf{F}(x, y, z) = \langle e^{xy}, e^{xz}, x^2 z \rangle$ ,  $S$  the portion of the ellipsoid  $4x^2 + y^2 + 4z^2 = 4$  to the right of the  $xz$ -plane
2. Use Stokes's Theorem to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  for  $C$  oriented counter clockwise from above.
  - (a)  $\mathbf{F}(x, y, z) = \langle x + y^2, y + z^2, z + x^2 \rangle$ ,  $C$  the triangle with vertices  $(1, 0, 0)$ ,  $(0, 1, 0)$ , and  $(0, 0, 1)$
  - (b)  $\mathbf{F}(x, y, z) = \langle 1, x + yz, xy - \sqrt{z} \rangle$ ,  $C$  the boundary of the plane  $3x + 2y + z = 1$  restricted to the first octant
  - (c)  $\mathbf{F}(x, y, z) = \langle xy, yz, zx \rangle$ ,  $C$  the boundary of the paraboloid  $z = 4 - x^2 - y^2$  intersect the first octant
  - (d)  $\mathbf{F}(x, y, z) = \langle 2y, xz, x + y \rangle$ ,  $C$  the intersection of the plane  $z = y + 2$  and the cylinder  $x^2 + y^2 = 1$
3. Verify Stokes's Theorem holds by computing  $\int_{\partial S} \mathbf{F} \cdot d\mathbf{r}$  and  $\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$  separately and comparing.
  - (a)  $\mathbf{F}(x, y, z) = \langle -y, x, -2 \rangle$ ,  $S$  the cone  $z^2 = x^2 + y^2$  with  $0 \leq z \leq 5$  oriented downward
  - (b)  $\mathbf{F}(x, y, z) = \langle -2yz, y, 3x \rangle$ ,  $S$  the part of the paraboloid  $z = 5 - x^2 - y^2$  above plane  $z = 1$  oriented upward
  - (c)  $\mathbf{F}(x, y, z) = \langle y, z, x \rangle$ ,  $S$  the hemisphere  $x^2 + y^2 + z^2 = 1$  with  $x \leq 0$  oriented in the negative  $x$ -direction
4. Let  $C$  be an arbitrary smooth, simple, closed curve in the plane  $x + y + z = 1$ . Show  $\int_C z \, dx - 2x \, dy + 3y \, dz$  depends only in the area enclosed by  $C$ .
5. Suppose  $S$  is a surface satisfying the hypotheses of Stokes's Theorem and  $f, g$  are functions with continuous second-order partial derivatives. Prove each of the following.
  - (a)  $\int_{\partial S} f \nabla g \cdot d\mathbf{r} = \iint_S (\nabla f \times \nabla g) \cdot d\mathbf{S}$
  - (b)  $\int_{\partial S} f \nabla f \cdot d\mathbf{r} = 0$
  - (c)  $\int_{\partial S} (f \nabla g + g \nabla f) \cdot d\mathbf{r} = 0$